ECL 4340

POWER SYSTEMS

LECTURE 12

POWER FLOWS, GAUSS-SEIDEL ITERATION

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ANNOUNCEMENTS

- Be reading Chapter 6, also Chapter 2.4 (Network Equations).
- HW 6 is posted. Due October 14, Friday, in Canvas.

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POWER FLOW ANALYSIS

- When analyzing power systems, we know neither the complex bus voltages nor the complex current injections
- Rather, we know the complex power being consumed by the load, and the power being injected by the generators plus their voltage magnitudes
- Therefore, we can not directly use the Y_{bus} equations, but rather must use the power balance equations

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Power Balance Equations

From KCL we know at each bus k in an N bus system the current injection, I_k, must be equal to the current that flows into the network

$$I_k = I_{Gk} - I_{Dk} = \sum_{n=1}^{N} I_{kn}$$

Since $\mathbf{I} = \mathbf{Y}_{\text{bus}}\mathbf{V}$ we also know

$$I_k = I_{Gk} - I_{Dk} = \sum_{n=1}^{N} Y_{kn} V_n$$

The network power injection is then $S_k = V_k I_k^*$

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Power Balance Equations

$$S_k = V_k I_k^* = V_k \left(\sum_{n=1}^{N} Y_{kn} V_n \right)^* = V_k \sum_{n=1}^{N} Y_{kn}^* V_n^*$$

This is an equation with complex numbers. Sometimes we would like an equivalent set of real power equations. These can be derived by defining

$$Y_{kn} \triangleq G_{kn} + jB_{kn} = |Y_{kn}| \angle \theta_{kn}$$

$$V_k \triangleq |V_k| e^{j\delta_k} = |V_k| \angle \delta_k$$

$$\delta_{kn} \triangleq \delta_k - \delta_n$$

Recall $e^{j\delta} = \cos\delta + j\sin\delta$

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Real Power Balance Equations

$$S_k = P_k + jQ_k = V_k \sum_{n=1}^{N} Y_{kn}^* V_n^* = \sum_{n=1}^{N} |V_k||V_n||Y_{kn}| e^{j(\delta_{kn} - \theta_{kn})}$$

$$= \sum_{n=1}^{N} |V_k||V_n||Y_{kn}| \angle \delta_{kn} - \theta_{kn}$$

Resolving into the real and imaginary parts

$$P_k = \sum_{n=1}^{N} |V_k||V_n||Y_{kn}| \cos(\delta_{kn} - \theta_{kn}) = P_{Gk} - P_{Dk}$$

$$Q_k = \sum_{k=1}^{N} |V_k||V_n||Y_{kn}| \sin(\delta_{kn} - \theta_{kn}) = Q_{Gk} - Q_{Dk}$$

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Real Power Balance Equations

$$S_k = P_k + jQ_k = V_k \sum_{n=1}^{N} Y_{kn}^* V_n^* = \sum_{n=1}^{N} |V_k||V_n| e^{j\delta_{kn}} (G_{kn} - jB_{kn})$$

$$= \sum_{n=1}^{N} |V_k||V_n| (\cos\delta_{kn} + j\sin\delta_{kn})(G_{kn} - jB_{kn})$$

Resolving into the real and imaginary parts

$$P_k = \sum_{n=1}^{N} |V_k||V_n|(G_{kn}\cos\delta_{kn} + B_{kn}\sin\delta_{kn}) = P_{Gk} - P_{Dk}$$

$$Q_k = \sum_{k=1}^{N} |V_k||V_n|(G_{kn}\sin\theta_{kn} - B_{kn}\cos\theta_{kn}) = Q_{Gk} - Q_{Dk}$$

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Power Flow Requires Iterative Solution

In the power flow we assume we know S_k and the $\mathbf{Y}_{\text{bus}}$. We would like to solve for the V's. The problem is the below equation has no closed-form solution:

$$S_k = V_k I_k^* = V_k \left(\sum_{n=1}^{N} Y_{kn} V_{kn} \right)^* = V_k \sum_{n=1}^{N} Y_{kn}^* V_n^*$$

Rather, we must pursue an iterative approach.

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Gauss Iteration

There are a number of different iterative methods we can use. We'll consider two: Gauss and Newton.

With the Gauss method we need to rewrite our equation in an implicit form: $x = h(x)$

To iterate we first make an initial guess of x, $x^{(0)}$, and then iteratively solve $x^{(v+1)} = h(x^{(v)})$ until we find a "fixed point", $\hat{x}$, such that $\hat{x} = h(\hat{x})$.

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Gauss Iteration Example

Example: Solve $x - \sqrt{x} - 1 = 0$

$$x^{(v+1)} = 1 + \sqrt{x^{(v)}}$$

Let $v = 0$ and arbitrarily guess $x^{(0)} = 1$ and solve

v	$x^{(v)}$	v	$x^{(v)}$
0	1	5	2.61185
1	2	6	2.61612
2	2.41421	7	2.61744
3	2.55538	8	2.61785
4	2.59805	9	2.61798

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Stopping Criteria

A key problem to address is when to stop the iteration. With the Guass iteration we stop when

$$\left|\Delta x^{(v)}\right| < \varepsilon \qquad \text{with } \Delta x^{(v)} \triangleq x^{(v+1)} - x^{(v)}$$

If x is a scalar this is clear, but if x is a vector we need to generalize the absolute value by using a norm

$$\left\|\Delta x^{(v)}\right\|_j < \varepsilon$$

Two common norms are the Euclidean & infinity

$$\|\Delta x\|_2 = \sqrt{\sum_{i=1}^{n} \Delta x_i^2} \qquad \|\Delta x\|_\infty = \max_i |\Delta x_i|$$

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Gauss Power Flow

We first need to put the equation in the correct form

$$S_k = V_k I_k^* = V_k \left(\sum_{n=1}^{N} Y_{kn} V_n \right)^* = V_k \sum_{n=1}^{N} Y_{kn}^* V_n^*$$

$$S_k^* = V_k^* I_k = V_k^* \sum_{n=1}^{N} Y_{kn} V_n$$

$$\frac{S_k^*}{V_k^*} = \sum_{n=1}^{N} Y_{kn} V_n = Y_{kk} V_k + \sum_{n=1, n \neq k}^{N} Y_{kn} V_n$$

$$V_k = \frac{1}{Y_{kk}} \left(\frac{S_i^*}{V_k^*} - \sum_{n=1, n \neq k}^{N} Y_{kn} V_n \right)$$

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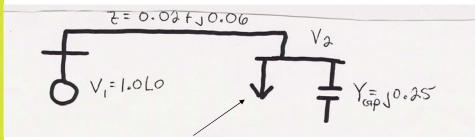
Gauss Two Bus Power Flow Example

A 100 MW, 50 Mvar load is connected to a generator through a line with $z = 0.02 + j0.06$ p.u. and line charging of 5 Mvar on each end (100 MVA base). Also, there is a 25 Mvar capacitor at bus 2. If the generator voltage is 1.0 p.u., what is V_2?



$S_{Load} = 1.0 + j0.5$ p.u.

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Gauss Two Bus Example, cont'd

The unknown is the complex load voltage, V_2.

To determine V_2 we need to know the $\mathbf{Y}_{bus}$.

$$\frac{1}{z} = \frac{1}{0.02 + j0.06} = 5 - j15$$

$$\text{Hence } \mathbf{Y}_{bus} = \begin{bmatrix} 5 - j14.95 & -5 + j15 \\ -5 + j15 & 5 - j14.70 \end{bmatrix}$$

$$(\text{Note } B_{22} = -j15 + j0.05 + j0.25)$$

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Gauss Two Bus Example, cont'd

$$V_2 = \frac{1}{Y_{22}} \left(\frac{S_2^*}{V_2^*} - \sum_{n=1, n \neq k}^{N} Y_{kn} V_n \right)$$

$$V_2 = \frac{1}{5 - j14.70} \left(\frac{-1 + j0.5}{V_2^*} - (-5 + j15)(1.0\angle 0) \right)$$

Guess $V_2^{(0)} = 1.0\angle 0$ (this is known as a flat start)

v	$V_2^{(v)}$	v	$V_2^{(v)}$
0	$1.000 + j0.000$	3	$0.9622 - j0.0556$
1	$0.9671 - j0.0568$	4	$0.9622 - j0.0556$
2	$0.9624 - j0.0553$		

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Gauss Two Bus Example, Cont'd

$V_2 = 0.9622 - j0.0556 = 0.9638\angle -3.3°$

Once the voltages are known all other values can be determined, such as the generator powers and the line flows

$S_1^* = V_1^*(Y_{11}V_1 + Y_{12}V_2) = 1.023 - j0.239$

In actual units $P_1 = 102.3$ MW, $Q_1 = 23.9$ Mvar

The capacitor is supplying $|V_2|^2\,25 = 23.2$ Mvar

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Slack Bus

- In previous example we specified S_2 and V_1 and then solved for S_1 and V_2.
- We can not arbitrarily specify S at all buses because total generation must equal total load + total losses
- We also need *an angle reference bus*.
- To solve these problems, we define one bus as the "slack" bus. This bus has a fixed voltage magnitude and angle, and a varying real/reactive power injection.

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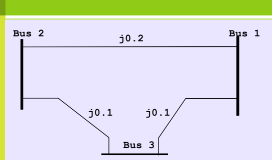
Stated Another Way



$$\mathbf{Y}_{bus} = j\begin{bmatrix} -15 & 5 & 10 \\ 5 & -15 & 10 \\ 10 & 10 & -20 \end{bmatrix}$$

$$\mathbf{V} = [\mathbf{Y}_{bus}]^{-1}\mathbf{I}$$

- This $\mathbf{Y}_{bus}$ is actually singular!
- So we cannot solve.
- This means (as you might expect), we cannot independently specify all the current injections **I**.

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Gauss with Many Bus Systems

With multiple bus systems we could calculate new V_k's as follows:

$$V_k^{(i+1)} = \frac{1}{Y_{kk}}\left(\frac{S_k^*}{V_k^{(i)*}} - \sum_{n=1,n\neq i}^{N} Y_{kn}V_n^{(i)}\right)$$
$$= h_i(V_1^{(i)}, V_2^{(i)}, \ldots, V_n^{(i)})$$

But after we've determined $V_k^{(i+1)}$ we have a better estimate of its voltage, so it makes sense to use this new value. This approach is known as the Gauss-Seidel iteration.

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Gauss-Seidel Iteration

Immediately use the new voltage estimates:

$$V_2^{(i+1)} = h_2(V_1, V_2^{(i)}, V_3^{(i)}, \ldots, V_N^{(i)})$$
$$V_3^{(i+1)} = h_2(V_1, V_2^{(i+1)}, V_3^{(i)}, \ldots, V_N^{(i)})$$
$$V_4^{(i+1)} = h_2(V_1, V_2^{(i+1)}, V_3^{(i+1)}, V_4^{(i)} \ldots, V_N^{(i)})$$
$$\vdots$$
$$V_n^{(i+1)} = h_2(V_1, V_2^{(i+1)}, V_3^{(i+1)}, V_4^{(i+1)} \ldots, V_N^{(i)})$$

The Gauss-Seidel works better than the Gauss, and is actually easier to implement. It is used instead of Gauss.

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Three Types of Power Flow Buses

- There are three main types of power flow buses
  - Load (PQ) at which P/Q are fixed; iteration solves for voltage magnitude and angle.
  - Slack at which the voltage magnitude and angle are fixed; iteration solves for P/Q injections
  - Generator (PV) at which P and |V| are fixed; iteration solves for voltage angle and Q injection
    - Special coding is needed to include PV buses in the Gauss-Seidel iteration

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Accelerated G-S Convergen

Previously in the Gauss-Seidel method we were calculating each value x as

$$x^{(i+1)} = h(x^{(i)})$$

To accelerate convergence we can rewrite this as

$$x^{(i+1)} = x^{(i)} + h(x^{(i)}) - x^{(i)}$$

Now introduce acceleration parameter α

$$x^{(i+1)} = x^{(i)} + \alpha(h(x^{(i)}) - x^{(i)})$$

With $\alpha = 1$ this is identical to standard Gauss-Seidel.

Larger values of α may result in faster convergence.

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Accelerated Convergence, cont'd

Consider the previous example: $x - \sqrt{x} - 1 = 0$

$$x^{(v+1)} = x^{(v)} + \alpha(1 + \sqrt{x^{(v)}} - x^{(v)})$$

Comparison of results with different values of α

k	$\alpha = 1$	$\alpha = 1.2$	$\alpha = 1.5$	$\alpha = 2$
0	1	1	1	1
1	2	2.20	2.5	3
2	2.4142	2.5399	2.6217	2.464
3	2.5554	2.6045	2.6179	2.675
4	2.5981	2.6157	2.6180	2.596
5	2.6118	2.6176	2.6180	2.626

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Gauss-Seidel Advantages/Disadvantage

- Advantages
  - Each iteration is relatively fast (computational order is proportional to number of branches + number of buses in the system
  - Relatively easy to program
- Disadvantages
  - Tends to converge relatively slowly, although this can be improved with acceleration
  - Has tendency to miss solutions, particularly on large systems
  - Tends to diverge on cases with negative branch reactances (common with compensated lines)
  - Need to program using complex numbers

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Power Flow

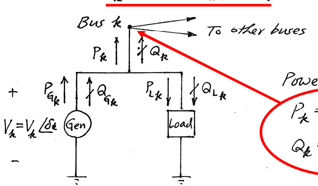
4. Power Flow Problem

(1) Bus Types:

The power-flow problem is to find voltage magnitude and phase angle at each bus in a balanced three-phase power system in steady state.

There are four variables in each bus:

V_k, δ_k, P_k, Q_k



Power Injection:

$P_k = P_{G_k} - P_{L_k}$

$Q_k = Q_{G_k} - Q_{L_k}$

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Power Flow

There are three types of buses:

1. Swing bus (slack bus) or (infinite bus): Reference bus. with $V_1 \angle \delta_1$, typically $1.0 \angle 0°$
   Needs to find P_1 and Q_1
2. Load bus (PQ bus). P and Q are specified
   Needs to find V_k and δ_k.
3. Voltage controlled bus (PV bus) or (Generator bus).
   P_k and V_k are specified
   Needs to find Q_k and δ_k

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Power Flow

Examples of PV bus:

Generators, switched shunt capacitors, static var systems.

- Upper and/or lower limits on Q are normally given: $Q_{G_{max}}$, $Q_{G_{min}}$
- If these limits are reached, then the reactive power output of the unit is held at the limit, and the bus is modeled as PQ bus (Load bus).
- Tap-changing transformer: Needs to compute the tap setting.

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Power Flow

(2) Network Equations:

$$I = Y_{bus} V$$

or

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1N} \\ Y_{21} & Y_{22} & & Y_{2N} \\ \vdots & & & \\ Y_{N1} & Y_{N2} & \cdots & Y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix}$$

current injection into each bus:

$$\Rightarrow \quad I_k = \sum_{n=1}^{N} Y_{kn} V_n$$

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Power Flow

(3) Power Equation:

Power injection at each bus is then,

$$S_k = P_k + jQ_k = V_k I_k^* \qquad I_k = \sum_{n=1}^{N} Y_{kn} V_n$$

$$= V_k \left[\sum_{n=1}^{N} Y_{kn} V_n \right]^*,$$

$$= |V_k| \sum_{n=1}^{N} |Y_{kn}||V_n| \, e^{j(\delta_k - \delta_n - \theta_{kn})}$$

where $V_n = |V_n| \angle \delta_n$, $Y_{kn} = |Y_{kn}| \angle \theta_{kn}$

Therefore,

$$\begin{cases} P_k = |V_k| \sum_{n=1}^{N} |Y_{kn}||V_n| \cos(\delta_k - \delta_n - \theta_{kn}) \\ Q_k = |V_k| \sum_{n=1}^{N} |Y_{kn}||V_n| \sin(\delta_k - \delta_n - \theta_{kn}) \end{cases}$$

$$k = 1, 2, \cdots, N$$

We only have $2N$ equations for $4N$ variables.

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Power Flow

Alternative Form: Y_{kn} in rectangular form,

$$Y_{kn} = G_{kn} + jB_{kn}$$

Then

$$\begin{cases} P_k = |V_k| \sum_{n=1}^{N} |V_n| \left[G_{kn} \cos(\delta_k - \delta_n) + B_{kn} \sin(\delta_k - \delta_n) \right] \\ Q_k = |V_k| \sum_{n=1}^{N} |V_n| \left[G_{kn} \sin(\delta_k - \delta_n) - B_{kn} \cos(\delta_k - \delta_n) \right] \end{cases}$$

These are "nonlinear" equations to solve.

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(2) Network Equations:

$$I = Y_{bus} V$$

or

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1N} \\ Y_{21} & Y_{22} & & Y_{2N} \\ \vdots & & & \\ Y_{N1} & Y_{N2} & \cdots & Y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix}$$

Power Flow

(4) Power-Flow Solution by Gauss-Seidel:

a) PQ bus: Use the network equation,

(1) $$I_k = \sum_{k=1}^{N} Y_{kn} V_n = Y_{k1} V_1 + \cdots + Y_{kk} V_k + \cdots + Y_{kN} V_N$$

with the current injection at bus k,

(2) $$I_k = \frac{P_k - jQ_k}{V_k^*}$$

which is resulting from the power injection,

$$S_k = P_k + jQ_k = V_k I_k^*$$

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Power Flow

(1) $$I_k = \sum_{k=1}^{N} Y_{kn} V_n = Y_{k1} V_1 + \cdots + Y_{kk} V_k + \cdots + Y_{kN} V_N$$

(2) $$I_k = \frac{P_k - jQ_k}{V_k^*}$$

Solving (1) for V_k,

$$V_k = \frac{1}{Y_{kk}} \left[\frac{P_k - jQ_k}{V_k^*} - \sum_{\substack{n=1 \\ n \neq k}}^{N} Y_{kn} V_n \right]$$

Applying Gauss-Seidel iteration,

(3) $$\underbrace{V_k^{(i+1)}}_{new} = \frac{1}{Y_{kk}} \left[\frac{P_k - jQ_k}{\underbrace{V_k^{*(i)}}_{old}} - \underbrace{\sum_{n=1}^{k-1} Y_{kn} V_n^{(i+1)}}_{Seidel} - \underbrace{\sum_{n=k+1}^{N} Y_{kn} V_n^{(i)}}_{Gauss} \right]$$

1. This can be repeated twice during each iteration, first with $V_k^{*(i)}$, then replacing it with $V_k^{*(i+1)}$.
2. Acceleration is performed to speed up the convergence,

$$V_{k,acc}^{(i+1)} = (1-\alpha) V_k^{(i)} + \alpha V_k^{(i+1)}$$
$$= V_k^{(i)} + \alpha \left(V_k^{(i+1)} - V_k^{(i)} \right), \quad \alpha \sim 1.6$$

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Therefore,

$$\begin{cases} P_k = |V_k| \sum_{n=1}^{N} |Y_{kn}| |V_n| \cos(\delta_k - \delta_n - \theta_{kn}) \\ Q_k = |V_k| \sum_{n=1}^{N} |Y_{kn}| |V_n| \sin(\delta_k - \delta_n - \theta_{kn}) \end{cases}$$

$k = 1, 2, \cdots, N$

Power Flow

b) PV bus: For a voltage controlled bus $|V_k|$ is specified.

1. We first calculate the corresponding reactive power with the power equation:

$$Q_k = |V_k| \sum_{n=1}^{N} |Y_{kn}| |V_n| \sin(\delta_k - \delta_n - \theta_{kn})$$

where $|V_n|$, $n = 1, 2, \cdots, N$, are the latest values available from the PQ buses.

or $$Q_k = -\mathrm{Im}\{ V_k^* I_k \}$$

$$S_k = P_k + jQ_k = V_k I_k^*$$

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Power Flow

2. We then check for the limit of the reactive power sources:

$$Q_{G_k} = Q_k + Q_{L_k}, \quad Q_{G_{min}} \le Q_{G_k} \le Q_{G_{max}}$$

where Q_{L_k} is the local load connected at bus k, and Q_{G_k} is the reactive power generation of the var source (Generator, SVC, etc.).

(i) If the limit is not violated, then use (3) to compute

$$V_k = |V_k| \angle \delta_k$$

Keep only δ_k, and define $V_k \triangleq |V_k|_{specified} \angle \delta_k$

(ii) If the limit is violated, then treat as PQ bus, with the Q_k set to the limit violated, i.e., $Q_k \triangleq Q_{G\,min\,or\,max} - Q_L$.

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Power Flow

c) <u>Swing bus:</u> V_1, δ_1 are specified.

After all PQ and PV bus power-flow calculations are converged, we compute P_1 and Q_1 at the last using the power equation.

Therefore,

$$\begin{cases} P_k = |V_k| \sum_{n=1}^{N} |Y_{kn}||V_n| \cos(\delta_k - \delta_n - \theta_{kn}) \\ Q_k = |V_k| \sum_{n=1}^{N} |Y_{kn}||V_n| \sin(\delta_k - \delta_n - \theta_{kn}) \end{cases}$$

$k = 1, 2, \cdots, N$

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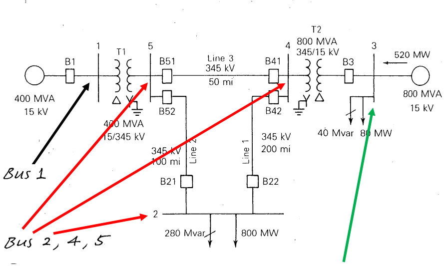
Power Flow

<u>Example 6.9</u>:



1. Swing bus: Bus 1

2. PQ bus: Bus 2, 4, 5

3. PV bus: Bus 3

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Power Flow

Input Data:

TABLE 6.1 Bus input data for Example 6.9*

Bus	Type	V per unit	δ degrees	P_G per unit	Q_G per unit	P_L per unit	Q_L per unit	Q_{Gmax} per unit	Q_{Gmin} per unit
1	Swing	1.0	0	—	—	0	0	—	—
2	Load	—	—	0	0	8.0	2.8	—	—
3	Constant voltage	1.05	—	5.2	—	0.8	0.4	4.0	−2.8
4	Load	—	—	0	0	0	0	—	—
5	Load	—	—	0	0	0	0	—	—

* S_{base} = 100 MVA, V_{base} = 15 kV at buses 1, 3, and 345 kV at buses 2, 4, 5

TABLE 6.2 Line input data for Example 6.9

Bus-to-Bus	R′ per unit	X′ per unit	G′ per unit	B′ per unit	Maximum MVA per unit
2–4	0.0090	0.100	0	1.72	12.0
2–5	0.0045	0.050	0	0.88	12.0
4–5	0.00225	0.025	0	0.44	12.0

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Power Flow

Input Data:

TABLE 6.3 Transformer input data for Example 6.9

Bus-to-Bus	R per unit	X per unit	G_c per unit	B_m per unit	Maximum MVA per unit	Maximum TAP Setting per unit
1–5	0.00150	0.02	0	0	6.0	—
3–4	0.00075	0.01	0	0	10.0	—

TABLE 6.4 Input data and unknowns for Example 6.9

Bus	Input Data	Unknowns
1	$V_1 = 1.0, \delta_1 = 0$	P_1, Q_1
2	$P_2 = P_{G2} - P_{L2} = -8$ $Q_2 = Q_{G2} - Q_{L2} = -2.8$	V_2, δ_2
3	$V_3 = 1.05$ $P_3 = P_{G3} - P_{L3} = 4.4$	Q_3, δ_3
4	$P_4 = 0, Q_4 = 0$	V_4, δ_4
5	$P_5 = 0, Q_5 = 0$	V_5, δ_5

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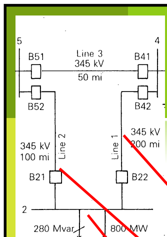

Bus-to-Bus	R′ per unit	X′ per unit	G′ per unit	B′ per unit
2–4	0.0090	0.100	0	1.72
2–5	0.0045	0.050	0	0.88
4–5	0.00225	0.025	0	0.44

The elements of Y_{bus} are computed from (6.4.2). Since buses 1 and 3 are not directly connected to bus 2,

$$Y_{21} = Y_{23} = 0$$

Using (6.4.2),

$$Y_{24} = \frac{-1}{R'_{24} + jX'_{24}} = \frac{-1}{0.009 + j0.1} = -0.89276 + j9.91964 \quad \text{per unit}$$

$$= 9.95972\angle 95.143^\circ \quad \text{per unit}$$

$$Y_{25} = \frac{-1}{R'_{25} + jX'_{25}} = \frac{-1}{0.0045 + j0.05} = -1.78552 + j19.83932 \quad \text{per unit}$$

$$= 19.9195\angle 95.143^\circ \quad \text{per unit}$$

$$Y_{22} = \frac{1}{R'_{24} + jX'_{24}} + \frac{1}{R'_{25} + jX'_{25}} + j\frac{B'_{24}}{2} + j\frac{B'_{25}}{2}$$

$$= (0.89276 - j9.91964) + (1.78552 - j19.83932) + j\frac{1.72}{2} + j\frac{0.88}{2}$$

$$= 2.67828 - j28.4590 = 28.5847\angle -84.624^\circ \quad \text{per unit}$$

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POWER FLOW

$$V_i = \frac{1}{Y_{ii}}\left(\frac{S_i^*}{V_i^*} - \sum_{k=1,k\neq i}^{n} Y_{ik}V_k\right)$$

$$V_2(1) = \frac{1}{Y_{22}}\left\{\frac{P_2 - jQ_2}{V_2^*(0)} - [Y_{21}V_1(1) + Y_{23}V_3(0) + Y_{24}V_4(0) + Y_{25}V_5(0)]\right\}$$

$$= \frac{1}{28.5847\angle{-84.624^\circ}}\left\{\frac{-8 - j(-2.8)}{1.0\angle{0^\circ}}\right.$$

$$\left. - [(-1.78552 + j19.83932)(1.0) + (-0.89276 + j9.91964)(1.0)]\right\}$$

$$= \frac{(-8 + j2.8) - (-2.67828 + j29.7589)}{28.5847\angle{-84.624^\circ}}$$

$$= 0.96132\angle{-16.543^\circ} \quad \text{per unit}$$

Next, the above value is used in (6.5.2) to recalculate $V_2(1)$:

$$V_2(1) = \frac{1}{28.5847\angle{-84.624^\circ}}\left\{\frac{-8 + j2.8}{0.96132\angle{16.543^\circ}}\right.$$

$$\left. - [-2.67828 + j29.75829]\right\}$$

$$= \frac{-4.4698 - j24.5973}{28.5847\angle{-84.624^\circ}} = 0.87460\angle{-15.675^\circ} \quad \text{per unit}$$

Computations are next performed at buses 3, 4, and 5 to complete the first Gauss–Seidel iteration.

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POWER FLOW

To see the complete convergence of this case, open PowerWorld Simulator case Example 6_10. By default, PowerWorld Simulator uses the Newton–Raphson method described in the next section. However, the case can be solved with the Gauss–Seidel approach by selecting **Simulation, Gauss–Seidel Power Flow**. To avoid getting stuck in an infinite loop if a case does not converge, PowerWorld Simulator places a limit on the maximum number of iterations. Usually for a Gauss–Seidel procedure this number is quite high, perhaps equal to 100 iterations. However, in this example to demonstrate the convergence characteristics of the Gauss–Seidel method it has been set to a single iteration, allowing the voltages to be viewed after each iteration. To step through the solution one iteration at a time, just repeatedly select **Simulation, Gauss–Seidel Power Flow**.

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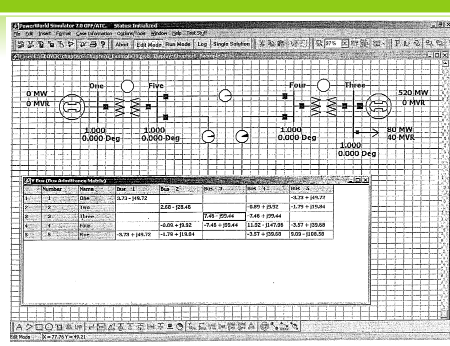
POWER FLOW



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